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Econometric-wavelet prediction in spatial aspect

Monika Hadaś-Dyduch

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Monika Hadaś-Dyduch
monika.dyduch@ue.katowice.pl
University of Economics in Katowice

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Abstract

The aim of this article is the prediction of GDP Polish and other selected European countries. For this purpose integrated into one algorithm econometric methods and wavelet analysis. Econometric methods and wavelet transform are combined goal of constructing a copyright model for predicting macroeconomic indicators. In the article, for estimating the macroeconomic indicators on the example of GDP proposed authorial algorithm that combines the following methods: a method trend creep method of alignment exponential and analysis multiresolution. Used econometric methods, this is a trend crawling and alignment exponential have been modified in several major stages. The aim of the merger of these methods is the construction of algorithm to predict short-term time series.

In the copyright algorithm was applied wavelet continuous compactly supported. wavelet used Daubechies. The Daubechies wavelets, are a family of orthogonal wavelets and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function which generates an orthogonal multiresolution analysis.

Keywords: *prediction, wavelets, wavelet transform*

JEL Classification: F37, C13, G15

1. Introduction

Economic forecasting is the process of making predictions about the economy. Forecasts can be carried out at a high level of aggregation or at a more disaggregated level, for specific sectors of the economy or even specific firms. The process of economic forecasting is similar to data analysis and results in estimated values for key economic variables in the future. An economist applies the techniques of econometrics in their forecasting process.

The aim of this article is a wavelet-econometric prediction. Described in the article, the algorithm is applied to the short-term prediction.

To research and predicting time series can be use a variety methods (see: Beyklin, 1992; Biernacki, 2007, 2009, 2012; Przybylska-Mazur, 2013).

Research shows (see. Bruzda, 2004; Hadaś-Dyduch, 2015a, 2015b, 2015c, 2016) that wavelet analysis can be used on a variety of academic levels, among other things: to study the properties of economic processes, smoothing ranks, removing noise, study the relationship between processes of different time scales and so on.

2. Wavelets

„Wavelets are functions that satisfy certain requirements. The very name wavelet comes from the requirement that they should integrate to zero, „waving“ above and below the x-axis. The diminutive connotation of wavelet suggest the function has to be well localized. Other requirements are technical and needed mostly to insure quick and easy calculation of the direct and inverse wavelet transform. There are many kinds of wavelets. One can choose between smooth wavelets, compactly supported wavelets, wavelets with simple mathematical expressions, wavelets with simple associated filters, etc.“ (Vidakovic and Mueller, 1994).

The most simple wavelet is the Haar wavelet. The technical disadvantage of the Haar wavelet is that it is not continuous, and therefore not differentiable. This property can, however, be an advantage for the analysis of signals with sudden transitions, such as monitoring of tool failure in machines (Lee and Tarng, 1999).

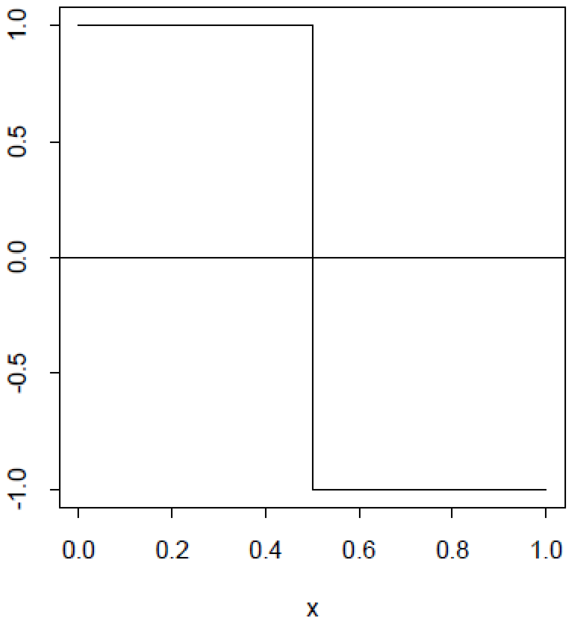

Fig. 1. Haar wavelet
Source: Own elaboration.

Apart from the the Haar wavelet there are many other types of waves, for example: Daubechies wavelets, Cohen-Daubechies-Feauveau wavelet, Mathieu wavelet, Legendre wavelet, Villasenor wavelet etc.

The Haar as a special case of the Daubechies wavelet, the Haar wavelet is also known as Db1. The Daubechies wavelets, based on the work of Ingrid Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis (see more in (Daubechies, 1992)).

3. Prediction algorithm

The article is an attempt to connect to into one algorithm econometric methods with wavelet analysis. Econometric methods and wavelet transform are combined for the construction of the model for prediction of time series.

The choice of econometric methods for prediction is wide. Distinguishing division of econometric methods due to the time, there are two groups, namely: static spatial information system and dynamic system of spatial information.

Static spatial information system, presents (usually constant, it is independent of the time) the relationship between the signal input and output circuit. Examination time characteristics in the context of the static does not make sense, because they do not say anything about a system in which there are no state variables. By contrast, dynamic spatial information system, is a kind of spatial information system, which in addition to the position of the object into account the time of its inception. This allows you to view the changes over time in the maps generated by the system. (Geographic information system, is a system of acquisition, processing and data sharing, which contain spatial information and accompanying descriptive information about the objects featured in the portion of the space covered by the operation of the system.).

In the article the author's design algorithm, numerous econometric methods, selected only adaptive methods. The difference between conventional methods and the methods of adaptation is as follows:

- Classical methods:
    - stimulus is often far from the threshold;
    - stimulus values to be presented are fixed before the experiment.
- Adaptive methods:
    - Modifications of the method of constant stimuli and method of limits;
    - Stimulus values to be presented depend critically on preceding responses.

Author's algorithm can be presented in several main stages as follows:

1) One-dimensional time series is divided into smaller, equinumerous unit, keeping the chronology of time.

2) For each series resulting from the division series in the base point 1), we determine the coefficients of wavelet Daubechies the following equations:

$$\begin{cases} \sum_{n=0}^{L-1} h_n = \sqrt{2} \\ \sum_{n=0}^{L-1} h_n h_{n+2m} = \delta_m \\ \sum_{q=0}^{L-1} q^k (-1)^k h_{L-1-q} = 1 \end{cases} \tag{1}$$

where:

L - filter length,

$$\delta_n = \begin{cases} 0 & for \quad n \neq 0 \\ 1 & for \quad n = 0 \end{cases} \tag{2}$$

It should be noted that signal processing using wavelet transform, uses filters. The filter h is called a low-pass filter, which is defined as (see more information in this regard in the monograph (Hadaś-Dyduch, 2015c)):

$$h_n = \langle \phi(x), \phi_{1,n}(x) \rangle \tag{3}$$

$$\phi(x) = \sqrt{2} \sum_{n \in Z} h_n \cdot \phi(2x - n) \tag{4}$$

3) Determine the function approximating to each series, according to the formula:

$$f(x) = \sum_{l \in Z} c_{j-1,l}\phi_{j-1,l}(x) + \sum_{l \in Z} d_{j-1,l}\psi_{j-1,l}(x) \tag{5}$$

where:

$$c_{j,n} = \left\langle f_j(x), \phi_{j,n}(x) \right\rangle \tag{6}$$

$$d_{j,n} = \left\langle f_j(x), \psi_{j,n}(x) \right\rangle \tag{7}$$

$$\psi(x) = \sqrt{2} \sum_{n \in Z} g_n \phi(2x - n) \tag{8}$$

4) Construction of models segmented according to the initial division of the unit series of base.
5) Determination of theoretical values, arising from the specific functions and series unit.
6) The calculation of the final value of the theoretical forecasted variable, according to the formula:

$$\hat{f}_t = \frac{1}{k_i} \sum_{j=1}^{k} \hat{f}_{ij}(t) \tag{9}$$

where:

$\hat{f}_{ij}(t)$ - final theoretical value for the period or the moment *t*,

k_i - the number of "segment" of theoretical variable values for the period or the moment t.

7) Solution of the problem:

$$Min\left\{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\left(\alpha\hat{y}_t + (1-\alpha)y_{t-1}\right) - y_t\right)^2}\right\} \tag{10}$$

on the assumption $\alpha \in \langle 0,1 \rangle$.

8) Prediction errors.

4. Research and results

Application copyright model for the production of short-term done for GDP, based on Eurostat data. The model described in chapter 3, was applied to the GDP recorded in the period 1995-2013. The model was applied to countries: Belgium, Bulgaria, Czech Republic, Poland. Based on the described algorithm determined GDP forecast for randomly selected countries, namely: Belgium, Bulgaria, Czech and Polish. The values obtained do not cover one hundred percent of real value, they are burdened with some errors. For alpha minimizing an error forecasts expired, prediction one period forward has the following errors:

- For Belgium: AE – 46.1; APE – 0.33%.
- For Bulgaria: AE – 44.4; APE – 0.31%.
- For the Czech Republic: AE – 39.4; APE – 0.29%.

- For Poland:AE – 48.4; APE – 0.38%.

The results of prediction are acceptable. Errors forecasts GDP selected countries obtained from a copyright prediction model are low compared with other prediction methods in the same category.

For study used Daubechies wavelet, however can also use other wavelets, such as: Meyer, Morlet, Haar or "Mexican hat". However, wavelet analyzes must have finite energy and the average value of zero. As a result takes the form of short-term oscillations.

Conclusion

The article presents a combination of the trend crawling with analysis of wavelet and forecasting using the method of alignment exponential. The results, presented prediction methods are relatively low compared with those obtained by other methods adaptation. It should be noted that the result of prediction has a significant impact applied method extensions sample during wavelet transform. The article uses the method of extension by zero, as the method gives the worst results. A significant impact on GDP, have also other factors, which are not included in the model (see: (Balcerzak and Pietrzak, 2016a, 2016b, 2016c, 2016d)). Among those factors there may be mentioned inter alia inflation (see: Przybylska-Mazur, 2012a, 2012b). The research in the article, can be extended to an additional aspect, it is the spatial aspect, for example using the methods described in (Balcerzak and Pietrzak, 2016e).

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