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# Model – spatial approach to prediction of minimum wage

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Abstract

The aim of the article is to present the author's model for prediction of the minimum wage. The model is based on wavelet analysis and methods of adaptation. Use of multiresolution analysis for prediction of the minimum wage in combination with the method to compensate the exponential gave good results in terms of minimizing the error. Research the minimum wage is very important. It should be noted that, in 2014, the level of gross minimum wages across the EU Member States varied from 33 % to just over 50 % of average gross monthly earnings for those persons working in industry, construction or services.

***Keywords:*** *wavelets, prediction, salary, minimum wage.*

***JEL Classification:*** C5, F3

***AMS Classification:*** 90C05

# 1 INTRODUCTON

In January 2016, 22 out of the 28 EU Member States (Denmark, Italy, Cyprus, Austria, Finland and Sweden were the exceptions) had a national minimum wage. As of 1 January 2016, monthly minimum wages varied widely, from EUR 215 in Bulgaria to EUR 1 923 in Luxembourg. There was also a national minimum wage in the following candidate countries of the EU: Albania, Montenegro, the former Yugoslav Republic of Macedonia, Serbia and Turkey. It should be noted that for those EU Member States outside of the euro area that have minimum wages (Bulgaria, the Czech Republic, Croatia, Hungary, Poland, Romania and the United Kingdom), as well as for Albania, the former Yugoslav Republic of Macedonia, Serbia, Turkey and the United States, the levels and ranking of minimum wages expressed in euro terms are affected by exchange rates. (see more in: Eurostat).

Research the minimum wage is very important. It should be noted that, in 2014, the level of gross minimum wages across the EU Member States varied from 33 % to just over 50 % of average gross monthly earnings for those persons working in industry, construction or services (activities of households as employers and extra-territorial organisations and bodies are excluded). The level of minimum wages in relation to the mean value of average gross monthly earnings was highest in Slovenia (51.3 %), Greece (50.1 %, 2011) and Turkey (50.0 %, 2010). At the lower end of the ranking, the United States (2013 data), the Czech Republic and Spain each reported that the level of their minimum wage was less than 35 % of average gross monthly earnings. (see: Eurostat).

The article discusses the problem of prediction work a minimum. It proposed an original model to predict the short-term, based on multiresolution analysis and econometric methods. To research and predicting time series can be use a variety methods (Biernacki 2009). Research shows (Hadaś-Dyduch 2015a, 2015b, 2015c, 2016; Vidakovic, Mueller 1994) that wavelet analysis can be used on a variety of academic levels, among other things: to study the properties of economic processes, smoothing ranks, removing noise, study the relationship between processes of different time scales and so on.

# 2 MULTIRESOLUTION ANALYSIS

A multiresolution analysis (MRA) or multiscale approximation (MSA) is the design method of most of the practically relevant discrete wavelet transforms (DWT) and the justification for the algorithm of the fast wavelet transform (FWT). It was introduced in this context in 1988/89 by Stephane Mallat and Yves Meyer and has predecessors in the microlocal analysis in the theory of differential equations (the ironing method) and the pyramid methods of image processing as introduced in 1981/83 by Peter J. Burt, Edward H. Adelson and James Crowley.

„Wavelets are functions that satisfy certain requirements. The very name wavelet comes from the requirement that they should integrate to zero, „waving“ above and below the x-axis. The diminutive connotation of wavelet suggest the function has to be well localized. Other requirements are technical and needed mostly to insure quick and easy calculation of the direct and inverse wavelet transform. There are many kinds of wavelets. One can choose between smooth wavelets, compactly supported wavelets, wavelets with simple mathematical expressions, wavelets with simple associated filters, etc.“ (Vidakovic, Mueller 1994). The most simple wavelet is the Haar wavelet. The Haar as a special case of the Daubechies wavelet, the Haar wavelet is also known as Db1. "In contrast to Haar’s simple-step wavelets, which exhibit jump discontiniues, Daubechies wavelets are continuous. As a consequence of their continuity, Daubechies wavelets approximate continuous signal more accurately with fewer wavelets than do Harr’s wavelets, but at the cost of intricate algorithms based upon a sophisticated theory. The Daubechies wavelets, are a family of orthogonal wavelets and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function which generates an orthogonal multiresolution analysis. Furthermore, each Daubechies wavelet is compactly supported. The Daubechies wavelets are neither symmetric nor antisymmetric around any axis, except for db1, which is in fact the Haar wavelet. Satisfying symmetry conditions cannot go together with all other properties of the Daubechies wavelets.” (Daubechies, 1992).

# 3 MODEL

The proposed algorithm for the prediction can be described as follows. In the first place, determined by the corresponding coefficients $a_k$, according to the following relationship::

$$a_k = \sum_{r=k+0}^{k+3} \varphi(r-k) p_r \, , \; k \in \{0,1,2,\ldots,2^n - 1\}. \tag{1}$$

where: $\varphi$ is a scaling function of Daubechies wavelet.

Followed by an application function approximating, the form (assuming that the initial number of the form: $p_0, p_1, \ldots, p_{2^n-2}, p_{2^n-1}$):

$$\begin{aligned}\tilde{f}(r) = a_{-2}\varphi(r+2) + a_{-1}\varphi(r+1) + a_0\varphi(r) + a_1\varphi(r-1) + a_2\varphi(r-2) \\ + \ldots + a_{2^{n-1}}\varphi\left(r - \left[2^n - 1\right]\right)\end{aligned} \tag{2}$$

where: $\varphi$ is a scaling function of Daubechies wavelet (Fig. 1).

The Daubechies wavelets, based on the work of Ingrid Daubechies, are a family of orthogonal wavelets defining a discrete wavelet transform and characterized by a maximal number of vanishing moments for some given support. With each wavelet type of this class, there is a scaling function (called the father wavelet) which generates an orthogonal multiresolution analysis.

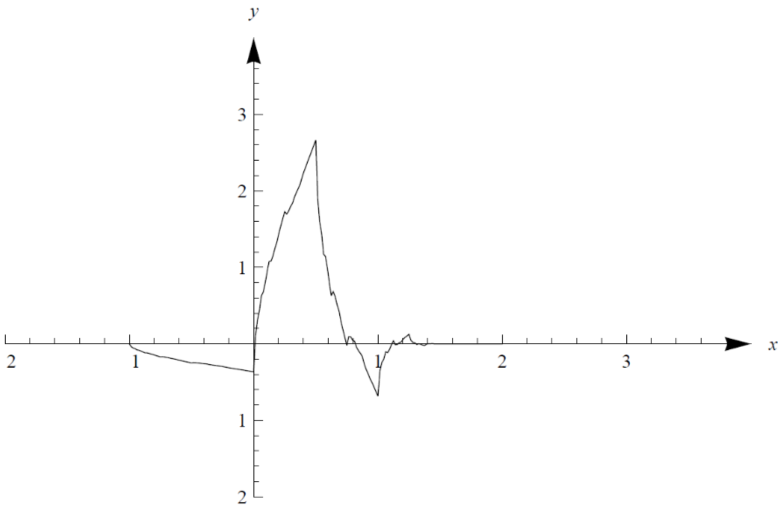


Fig. 1. **Daubechies wavelet.**

Source: Own elaboration.

Using Daubechies wavelet functions

$$\varphi^{p}_{D,M_n}(x) = 2^{\frac{M}{2}} \varphi^{p}_{D}\left(2^{M} x - n\right), \quad \psi^{p}_{D,mn}(x) = 2^{\frac{m}{2}} \psi^{p}_{D}\left(2^{m} x - n\right)$$

and given certain conditions (see: Hasiewcz, Śliwiński 2005), we get the following models decomposed wavelet (see also: Hasiewcz, Śliwiński 2005):

$$\hat{g}^{p}_{D}(x,K) = \sum_{n=\left[2^{M} x\right]-2p+1}^{\left[2^{M} x\right]} \hat{\alpha}^{p,g}_{D,Mn} \varphi^{p}_{D}\left(2^{M} x - n\right) + \sum_{m=M}^{K-1} \sum_{n=\left[2^{M} x\right]-p}^{\left[2^{M} x\right]} \hat{\beta}^{p,g}_{D,mn} \psi^{p}_{D}\left(2^{m} x - n\right)$$

$$\hat{f}^{p}_{D}(x,K) = \sum_{n=\left[2^{M} x\right]-2p+1}^{\left[2^{M} x\right]} \hat{\alpha}^{p,f}_{D,Mn} \varphi^{p}_{D}\left(2^{M} x - n\right) + \sum_{m=M}^{K-1} \sum_{n=\left[2^{M} x\right]-p}^{\left[2^{M} x\right]} \hat{\beta}^{p,f}_{D,mn} \psi^{p}_{D}\left(2^{m} x - n\right)$$

where:

$$\hat{\alpha}^{p,g}_{D,Mn} = 2^M \sum_{\{k:u_{Mn,k}\in[0,2p-1]\}} y_k \varphi^p_D\left(u_{Mn.,k}\right),$$

$$\hat{\alpha}^{p,f}_{D,Mn} = 2^M \sum_{\{k:u_{Mn,k}\in[0,2p-1]\}} \varphi^p_D\left(u_{Mn.,k}\right)$$

$$\hat{\beta}^{p,g}_{D,Mn} = 2^m \sum_{\{k:u_{mn,k}\in[1-p,p]\}} y_k \psi^p_D\left(u_{mn.,k}\right),\ u_{mn,k} = 2^m x_k - n\,,$$

$$\hat{\beta}^{p,g}_{D,Mn} = 2^m \sum_{\{k:u_{mn,k}\in[1-p,p]\}} \psi^p_D\left(u_{mn.,k}\right)$$

$\varphi$ - is a scaling function of Daubechies wavelet,

$$\int_{-\infty}^{+\infty} \varphi(x)dx = 1\,,$$

$\psi$ - is a Daubechies wavelet, $\int_{-\infty}^{+\infty} \psi(x)dx = 0$

Featuring a smoothed time series, which aim to simplify, written as: $\hat{y}_1, \hat{y}_2, \hat{y}_3, \ldots, \hat{y}_n$, undertakes to solution a simple task:

$$Min\left\{\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\left(\alpha\hat{y}_t + (1-\alpha)y_{t-1}\right) - y_t\right)^2}\right\},$$

$$\alpha \in \langle 0,1 \rangle \qquad (3)$$

The forecast for one period forward is determined by the formula: $\hat{\hat{y}}^P_{t+1} = \alpha \cdot \hat{y}_t + (1-\alpha)\cdot y_t$. Wherein $\hat{y}_t$ value smoothed by the trend creepy-wavelet, and parameter $\alpha \in [0, 1]$ - called the smoothing constant, finessed so as to minimize errors *ex-post* prediction.

# 4 RESULT

Application copyright model for the production of short-term done for monthly minimum wages - bi-annual data, based on Eurostat data. Data on gross monthly earnings, which is based on the study, cover remuneration in cash paid before any tax deductions and social security contributions payable by wage earners and retained by the employer, and restricted to gross earnings which are paid in each pay period. (see: Eurostat). The model described in chapter 3, were applied to the monthly minimum wages - bi-annual data. The model was applied to countries: Belgium, Bulgaria, Czech Republic, Estonia, Ireland, Greece.

Based on the described algorithm determined wages forecast for randomly selected countries, namely: Belgium, Bulgaria, Czech Republic, Estonia, Ireland, Greece. The values obtained do not cover one hundred percent of real value, they are burdened with some errors. For alpha minimizing an error forecasts expired, prediction one period forward has the following errors:

- For Belgium: APE – 0,35%.
- For Bulgaria: APE – 0,33%.
- For the Czech Republic: APE – 0,39%.
- For Estonia: APE – 0,44%.
- For Ireland: APE – 0,39%.
- For Greece: APE – 0,42%.

The results of prediction are acceptable. Errors forecasts wages selected countries obtained from a copyright prediction model are low compared with other prediction methods in the same category.

# 5 CONCLUSION

Research the minimum wage is an important issue for a substantial part of the population. The proportion of workers whose earnings are equal to the minimum wage is very different depending on the country. Studies show that, I n 2014, the level of gross minimum wages across the EU Member States varied from 33 % to just over 50 % of average gross monthly earnings for those persons working in industry, construction or services (activities of households as employers and extra-territorial organisations and bodies are excluded) as covered by NACE Rev. 2 Sections B–S (NACE - The Statistical classification of economic activities in the European Community, abbreviated as NACE, is the classification of economic activities in the European Union (EU); the term NACE is derived from the French Nomenclature statistique des activités économiques dans la Communauté européenne. Various NACE versions have been developed since 1970. NACE is a four-digit classification providing the framework for collecting and presenting a large range of statistical data according to economic activity in the fields of economic statistics (e.g. production, employment and national accounts) and in other statistical domains developed within the European statistical system (ESS). NACE Rev. 2, a revised classification, was adopted at the end of 2006 and, in 2007, its implementation began. The first reference year for NACE Rev. 2

compatible statistics is 2008, after which NACE Rev. 2 will be consistently applied to all relevant statistical domains (Eurostat)). The minimum wage is related to investments and savings. It is worth in this context, to analyze the position of [Hadaś-Dyduch, 2014].

Research the minimum wage, conducted for the application model prediction. The test can be conducted from a different perspective, a more spatial, in a manner clearly different, eg. as presented in research (Balcerzak, Pietrzak 2015a, 2015b, 2016a, 2016b).

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